when $b = 0$, the value of Δt is

$$\Delta t = \frac{\bar{\rho}_s I}{S^2 c\rho}(\Delta I\theta) \qquad (22)$$

in this case Δt will be uniform over the length of the wire and there will be no longitudinal heat flux. When the change in Δt is positive there will be a longitudinal heat flux from high ϑ_0 towards low ϑ_0, that is from the film boiling zone towards the nucleate boiling zone, when b is negative the flux will be in the other direction from the nucleate toward the film boiling zone, when $b = 0$ there will be no tendency from the zones to change in extent.

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The photograph of coexistent boiling has been taken in Dr. van Stralen's laboratory.

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LEADING EDGE EFFECTS ON THE NUSSELT NUMBER FOR A VERTICAL PLATE IN FREE CONVECTION

J. GRYZAGORIDIS

Mechanical Engineering Department, University of Cape Town, Cape Town, S. Africa

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NOMENCLATURE

- C_p, specific heat of the fluid;
- g, acceleration due to gravity;
- Gr_x, local Grashof number $\rho^2\beta g(T_w - T_\infty)\,x^3/\mu^2$;
- h_c, free convective coefficient;
- k, thermal conductivity of the fluid;
- L, width of the plate;
- Nu_x, local Nusselt number $h_c x/k$;
- Pr, Prandtl number $C_p\mu/k$;
- q, heat transfer per unit area;
- T, temperature;
- ΔT, temperature difference;
- ΔN, fringe shift;
- n, refractive index of fluid;
- x, distance along the length of the plate;
- y, distance perpendicular to the plate.

Greek symbols

- β, coefficient of thermal expansion of the fluid;
- λ, wavelength of light;
- μ, absolute viscosity of the fluid;
- ρ, density of the fluid.

Subscripts

- w, wall conditions;
- ∞, undisturbed fluid;
- L, based on overall height of the plate.

INTRODUCTION

It is to be expected that the classical boundary layer solution by Schmidt and Beckmann [1] for local heat transfer rates from a vertical plate in free convection

$$Nu_x = 0{\cdot}360\,(Gr_x)^{\frac{1}{4}} \qquad (1)$$

and the integral method by Eckert and Drake [2] which yielded

$$Nu_x = 0{\cdot}378\,(Gr_x)^{\frac{1}{4}} \qquad (2)$$

are not adequate to explain the phenomenon of free convection from a vertical plate, near the leading edge, since they do not account for the flow that starts ahead of the plate which some investigators call induced flow.

Previous papers by Suriano and Yang [3], Yang and Jerger [4], and Suriano Young and Donlon [5], have dealt with this aspect using perturbation techniques or finite difference schemes and they have shown departure from the classical solution approximately below $Gr \simeq 10^3$.

More recently Cheesewright [6], has shown experimental results in agreement with the classical solution as low as $Gr_x = 10^3$. The author [7], has shown that when average heat transfer rates are calculated, the empirical power law relationship

$$Nu_L = 0{\cdot}555\,(Gr_L Pr)^{\frac{1}{4}} \tag{3}$$

holds as low as $Gr \simeq 10$, the reason being that the leading edge effects which take place in a relatively small percentage of the boundary layer exhibited around the whole plate, are lost in the integration over the entire length of the plate. It appears that detailed experimental data near the leading edge of a vertical plate are very much needed as far as local values are concerned.

EXPERIMENTAL TECHNIQUE

In order to obtain local Nusselt numbers near the leading edge of a verical plate, the author constructed a 10 cm field Mach–Zehnder Interferometer, where an electrically heated plate, having the dimensions: 6·35 cm height, 10·15 cm width, 0·5 cm thickness, was placed for observation. From the interferograms taken local Nusselt numbers were evaluated using the following technique.

The interferometer was adjusted for wedge fringes field and when steady state conditions were established for the plate, a photographic record was taken. The temperature of the plate was evaluated using the following equation which relates the fluid temperature change ΔT to the fringe shift ΔN, length of the model L, wavelength of light λ, refractive index of air n, and the undisturbed fluid temperature T_∞,

$$\Delta T = \frac{T_\infty \Delta N}{(L/\lambda)(n-1) - \Delta N}. \tag{4}$$

As a check measure the plate was equipped with six thermocouples spaced 1·0 cm apart from the leading edge. In each case the temperature of the plate calculated from equation (4) was identical to the one indicated by the corresponding thermocouple. With the wedge fringes oriented perpendicular to the plate, the plate temperature was indicated by the displacement of the fringe from its initial undisturbed position to where it met the surface. The slope of the fringe near the surface is directly related to the heat loss from the surface by convection thus there is no need to account for radiation losses, and since the plate was suspended by thin wires there were negligible conduction losses. The heat flow from the surface was calculated using the conduction equation

$$q = -k \left.\frac{\mathrm{d}T}{\mathrm{d}y}\right|_{y=0}. \tag{5}$$

The values of the temperature gradients at the surface were obtained from the interferograms for any desired distance from the leading edge and the thermal conductivity of the fluid was evaluated at the surface temperature.

The heat loss from a surface by convection is usually expressed by the relation,

$$q = h_c(T_w - T_\infty) \tag{6}$$

and from the above the local free convective coefficients were evaluated and subsequently the local Nusselt numbers. The corresponding Grashof numbers were evaluated at the film temperature,

$$T_f = \frac{T_w + T_\infty}{2}. \tag{7}$$

DISCUSSION

The results of this investigation show excellent agreement with equations (1) and (2) away from the leading edge, i.e. at high Grashof numbers, and confirm previous data reported by Cheesewright [6]. However as expected, the results show conclusively that the range of applicability of boundary layer concepts, as far as *local* values are concerned, is for Grashof numbers greater than 10^4. In Fig. 1 it is seen that below $Gr_x \simeq 5 \times 10^3$ there is a marked departure from the accepted expressions thus showing that the *local* Grashof–Nusselt number relationship is not universal at low Grashof numbers obtained by decreasing coordinate along the plate toward the leading edge.

When performing experiments on a vertical plate in air, for all practical purposes low Grashof numbers may be obtained, by small ΔT's, reduced densities, or using small plates. With the first two methods i.e. $\Delta T \to 0$ and $\rho \to 0$, the convection currents decay thus approaching the pure conduction limit, however when small Grashof numbers are approached by decreasing coordinate toward the leading edge, the pure conduction limit is not approached, the reason being that the boundary layer starts ahead of the leading edge.

The author has reported data in [7] for *average* heat transfer values for similar size plates with the one used in the present investigation, obtained by letting $\rho \to 0$, showing that the accepted power law relationship

$$Nu_L = 0{\cdot}555\,(Gr_L Pr)^{\frac{1}{4}}$$

is adequate for the range of Grashof numbers between 10 and 10^8. For *average* values the leading edge effects which take place in a relatively small percentage of the plate are not significant because they are lost in the integration process.

In the past, investigators have recognized the need of refining the boundary layer solution in order to describe the phenomenon more accurately in the region of finite Grashof numbers. To the knowledge of this writer the first attempt was presented by Yang and Jerger [4]. Their results however are in variance with the data reported in this investigation. This is because, in their first order perturbation analysis, they have failed to take into account the leading edge effects. It appears that these are quite significant, and they are mainly due to the induced flow ahead of the leading edge.

Suriano Yang and Donlon [5] presented a paper where they have shown that the corrections on the boundary layer values due to the leading edge effects are much more significant than due to the boundary layer perturbations. Unfortunately their results are presented as average values starting about $Gr_L = 10^4$ and do not permit direct comparison with the present data, reported in Fig. 1, however they do indicate similar departure from the accepted solution.

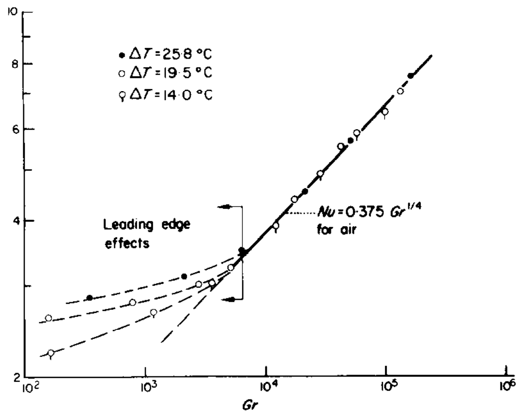

FIG. 1. Local heat transfer rates from a vertical plate in free convection showing the effects of leading edge.

An interesting fact comes to light examining the points of departure in Fig. 1. The departure occured at approximately the same distance from the leading edge (12·7 mm), but due to different temperature differences, at different Grashof numbers. In other words as the overall temperature difference ΔT, decreased for the same distance from the leading edge, the Grashof number became smaller and the agreement with the theory was extended!

The exact mechanism of how the flow starts ahead of the leading edge is not known, however professor Gebhart's discussion in [4] on the behavior of the flow in the immediate neighborhood of the leading edge is well taken and worthwhile quoting here. "This flow is induced by lower pressures which are found near the leading edge due to the presence of the normal component of velocity v_∞ resulting from the buoyancy induced flow. Since v_∞ decreases toward zero with increasing x, the pressure outside the boundary region (excepting hydrostatic effects) increases with x toward the limiting value in the completely undisturbed fluid, say p_∞, thus inducing a flow field outside of the boundary region."

Bearing in mind the above mentioned the tendency of the author's data to agree with the theory at lower temperature differences may be explained. Because the temperature differences are smaller, the buoyancy induced flow is less, and consequently the effects of the induced flow ahead of the leading edge are reduced (see Figs. 2–4). It appears from the data reported here, that there is a surprising influence of the temperature difference on the Nusselt number for fixed

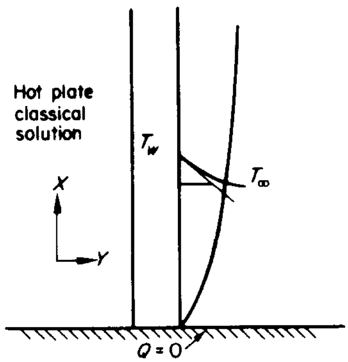

FIG. 2.

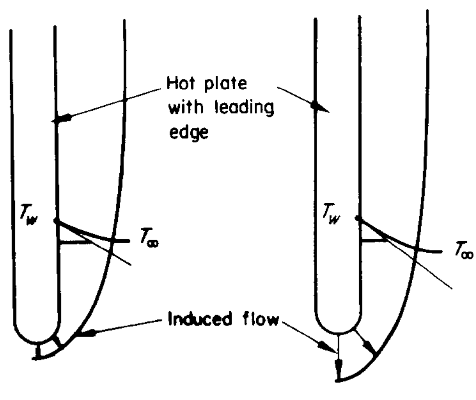

FIG. 3. FIG. 4.

Grashof numbers, in the region below 5×10^3. This to the knowledge of the author is new data which lead to a new set of questions. If the temperature difference is a new parameter as shown in Fig. 1, then there must be a dimensionless parameter associated with this temperature difference. What could that be? Can this parameter be extracted out of the complete differential equations? In the table below some of the experimental data is provided so that interested readers may draw their own conclusions.

Table 1.

x (mm)	Nu_x	Gr_x	ΔT (°C)
25·4	4·8	$2{\cdot}98 \times 10^4$	14·0
12·7	3·1	$3{\cdot}72 \times 10^3$	14·0
8·75	2·66	$1{\cdot}2 \times 10^3$	14·0
4·60	2·22	$1{\cdot}66 \times 10^2$	14·0
12·7	3·2	$5{\cdot}2 \times 10^3$	19·5
11·1	2·95	$2{\cdot}76 \times 10^3$	19·5
6·8	2·77	$8{\cdot}0 \times 10^2$	19·5
3·95	2·62	$1{\cdot}58 \times 10^2$	19·5
12·7	3·5	$6{\cdot}46 \times 10^3$	25·8
8·75	3·1	$2{\cdot}1 \times 10^3$	25·8
4·75	2·82	$3{\cdot}42 \times 10^2$	25·8

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INERT-GAS TRANSPORT IN LIQUID METALS DURING BOILING EXPERIMENTS

ROBERT E. HOLTZ, HANS K. FAUSKE and DONALD T. EGGEN*

Argonne National Laboratory, Argonne, Illinois, U.S.A.

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NOMENCLATURE

C_A, molar concentration of inert gas in liquid solution;
C_{A1}, molar concentration of inert gas in liquid solution at $t \geqq 0$ and $X = X_2$;
C'_A, molar concentration of inert gas in surface cavities;
C_B, molar concentration of liquid in liquid solution;
C'_B, molar concentration of vapor in surface cavities;
D_{AB}, mass diffusivity coefficient of inert gas in liquid solution;
D'_{AB}, mass diffusivity coefficient of inert gas in vapor;
K, Henry's Law constant;
M_B, molecular weight of liquid;
P_A, partial pressure of inert gas in surface cavity;
P_{A1}, P_{AG}, partial pressure of inert gas in gas blanket;
$P_A(t)$, partial pressure of inert gas in surface cavities at time t;
P_{gb}, pressure at liquid–inert gas blanket interface;
T, liquid temperature;
T_{gb}, temperature of liquid at inert-gas blanket–liquid interface;
T_0, initial liquid temperature;
t, time;
V_A, molar volume of A in liquid at normal boiling point;
X, coordinate in isothermal system;
X_A, mole fraction of inert gas in liquid solution;
X_{gb}, mole fraction of inert gas in liquid sodium at blanket–liquid interface;
X_3, liquid metal pool depth;

* With Northwestern University, Evanston, Illinois.